Abstract

- Proposed a Euclidean Distance Transform (EDT) based skeletonization method to find the midline of the vessel.
- Provide a convenient method to measure the vessel diameter using the midline from the images.

Calculation of the Midline

- **Make Binary Image**
  Automatically find the global threshold $T$, change pixels with gray level values more than $T$ (object points) into 255 and pixels with values not more than $T$ (background points) into 0.

- **Euclidean Distance Transform (EDT)**
  
  \[
  D_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}
  \]

  $O(x,y)$ is an object point in the image, $E$ is the Euclidean Distance value of the point $O$, $N(x,y)$ is the nearest boundary point of $O$.

- **Calculate the Gradient Vector Field of the EDT Map**
  After getting the distance transform of the image, we can treat the distance map as a mountain where the skeleton of the image is just the ridge of the mountain. $Z_1, Z_2, Z_3$ are nine points in a $3 \times 3$ filter with $Z_1$ in the center, the gradient value of object point $O(i,j)$ is defined as:
  
  \[
  \text{grad}(i,j) = 4 \times \text{grad}(i,j)'
  \]

- **Find the Maximum of the EDT Map, Critical Points and Gradient Path of each Points**
  Name the point with the maximum of the EDT as $\text{MAX}$. Critical points: object points with local maximal distance value and minimum gradient value. $P = \{P_1, P_2, \cdots , P_n\}$ is an 8-connected path, the gradient length is defined as:
  
  \[
  L = \sum_{ij} |\text{grad}(i,j)|
  \]

  Gradient distance of two points $r_1$ and $r_2$ is defined as minimum over the gradient lengths of all 8-connected paths joining them. Gradient path: the 8-connected path with the smallest gradient distance.

  All object points $O(i,j)$ has a data structure $(D_{ij}, P_{ij})$. $D_{ij}$ is the gradient distance value between $O(i,j)$ and $\text{MAX}$, $P_{ij}$ is the coordinates of the point before $O(i,j)$ along the gradient path (i.e. the parent point).

  Set $D(\text{MAX}) = 0, P(\text{MAX}) = \text{null}$.

  Divide the image into four quadrants with $\text{MAX}$ as the origin. Scanning from $\text{MAX}$ to four corners of the image. For each point $O(i,j)$:
  
  \[
  D(i,j) = \min(D1, D2, \cdots , D8) + \text{grad}(i,j)
  \]

  \[
  P(i,j) = P_{ij}
  \]

  $P(i,j)$ is the neighbors of $O(i,j)$ with the smallest gradient value. Then scan the whole image from four corners to their diagonal corners. Each corner will scan twice, beginning with two different directions.

- **Connect the Skeleton Points**
  1. Find the critical point that has the largest $D$. Use KEY to record its coordinate.
  2. If $P(\text{KEY}) = \text{MAX}$, go to 4; else if $\text{KEY}$ is critical point, remove it by doing $\text{Criticalpoint}[\text{KEY}] = \text{false}$.
  3. $\text{Skeleton}[\text{KEY}] = \text{true};$ KEY = P(\text{KEY}); go back to 2
  4. If there is no critical point, stop; else go to 1

Boundary Extracting

The boundary of a set $A$, denoted by $\partial(A)$, can be obtained by first eroding $A$ by a suitable structuring element $B$ and then performing the set difference between $A$ and its erosion $\rho(A) = A - (A \ominus B)$.

Calculation of the Vessel Diameter

- **Slope calculation**
  Use liner regression to calculate the slope of a given skeleton point $(x,y)$
  
  \[
  b = \frac{\left( \sum x_i \sum x_i \right) - m \left( \sum x_i y_i \right)}{\left( \sum x_i \right)^2 - m \left( \sum x_i^2 \right)}
  \]

  where $(x_i, y_i), i=1,2,\ldots,m$ are the neighbor skeleton points of $O(x,y)$.

  Since the screen uses left hand coordinates, the slope $k'$ on the screen should be $b' = -1/k'$ and the slope of the perpendicular line will be $k = -1/k'$.  

- **Distance calculation**
  The diameter of vessel at the skeleton point $(x_0,y_0)$ is
  
  \[
  d = \sqrt{(x_0-x_1)^2 + (y_0-y_1)^2} 
  \]

  Line $x = ax + by + c$ intersects with border on $(x_1,y_1)$ and $(x_2,y_2)$. Real vessel diameter is $d = \text{display resolution} \times k' \sqrt{a^2 + b^2}$ where $k'$ depends on the geometry of the ultrasound scanning and the screen’s display resolution.

Conclusion

We have proposed an algorithm to track the blood vessel in 3D ultrasound power-mode image, followed by a technique to measure the vessel diameter interactively. The main reason we used power-mode image is that it offers automatic object segmentation, i.e., the power-mode image only shows the blood flow information inside vessel structure with no information displayed for surrounding tissue.